Circle Coordinate Geometry MS

1.

Marking Instructions	AO	Marks	Typical Solution
Selects correct answer	AO1.1b	B1	$\frac{2}{3}$
Total		**1**	

2.

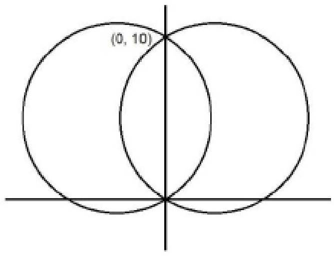
Circles correct answer	AO1.1b	B1	9π
Total		**1**	

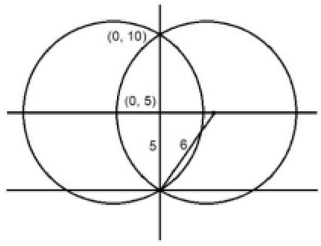
3.

1(a)(i)	States correct radius CAO	AO1.2	B1	Radius $=\sqrt{5}$
(a)(ii)	States correct centre CAO	AO1.2	B1	C is $(7, -2)$
(b)	Finds gradient of the line through the points P and 'their' C (as found in part **(a)**) Condone one sign error	AO3.1a	M1	Gradient $CP = \frac{-1-(-2)}{5-7} = -\frac{1}{2}$
	Correct tangent gradient obtained from 'their' CP gradient	AO3.1a	M1	So tangent gradient = 2
	Uses a correct form for the equation of a straight line with correct coordinates of P and 'their' tangent gradient	AO1.1a	M1	$y-(-1) = 2(x-5)$
	States correct final answer in required form ($y = mx + c$) FT from 'their' C found in part **(a)**	AO1.1b	A1F	$y = 2x - 11$

(c)	Identifies QTC as a right-angled triangle PI	AO3.1a	M1	QTC is a right-angled triangle so we can use Pythagoras
	Finds QC or QC^2 FT 'their' C found in part **(a)**	AO1.1b	B1F	$QC^2 = (7 - 3)^2 + (-2-3)^2$
	Uses Pythagoras' theorem correctly for 'their' triangle	AO1.1a	M1	$4^2 + 5^2 = (\sqrt{5})^2 + QT^2$
	Correct evaluation of length of QT FT 'their' QC and 'their' radius found in part **(a)**	AO1.1b	A1F	$QT^2 = 36$ so $QT = 6$
	Total		**10**	

4.

(a)	Produces a combined diagram showing circles intersecting at origin and (0, 10) or two separate diagrams. Allow reasonable 'hand drawn' circles which illustrate symmetry. Circles must cut the x axis again. Do not accept circles that go off the page.	AO2.2a	B1	

(b)	Deduces that y coordinate of centre is 5. (PI by any use of $(y-5)$ in any circle equation or marked on diagram or seen as a y coordinate or used in Pythagoras)	AO2.2a	B1	
	Forms correct equation for x coordinate of centres using Pythagoras (PI)	AO1.1a	M1	$6^2 = 5^2 + a^2$ $a = +\sqrt{11}$ or $-\sqrt{11}$
	Obtains two correct circle equations (either form) Condone 3.3 or better provided $a = \sqrt{11}$ seen earlier	AO1.1b	A1	$(x-\sqrt{11})^2 + (y-5)^2 = 6^2 = 36$ $(x+\sqrt{11})^2 + (y-5)^2 = 6^2 = 36$ Or $x^2 + 2\sqrt{11}x + y^2 - 10y = 0$ $x^2 - 2\sqrt{11}x + y^2 - 10y = 0$
	Total		**4**	

5.

(a)	Completes the square twice or applies standard formula	AO1.1a	M1	$(x+4)^2 + (y-6)^2 - 16 - 36 = 12$ $(x+4)^2 + (y-6)^2 = 64$
	Obtains correct equation	AO1.1b	A1	Centre $(-4, 6)$ Radius = 8
	Obtains correct radius and correct coordinates of C Follow through 'their' equation	AO1.1b	A1F	

(b)	Demonstrates a method to find the length OP or OQ (or their squares), or the coordinates P or Q using 'their' values from part **(a)**	AO3.1a	M1	$OC^2 = 4^2 + 6^2 = 52$ $OP^2 = r^2 - OC^2$ $= 64 - 52 = 12$ $PQ = 2OP$ $= 2\sqrt{12} = 4\sqrt{3}$
	Uses a circle property that may lead to a solution, eg radius and chord meet at right-angles (evidence for this could be the use of Pythagoras or perpendicular gradients)	AO3.1a	M1	
	Finds OP or OQ or coordinates of P or Q **CAO**	AO1.1b	A1	
	Obtains length of PQ Follow through from 'their' coordinate of P and Q (Does not need to be in the required form)	AO1.1b	A1F	
	Sets out a well-constructed mathematical argument, using precise statements and correct use of symbols throughout to show the correct required result in required form Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips	AO2.1	R1	
	Total		**8**	

6.

(a)	Completes the square twice or applies standard formula for C_1	AO3.1a	M1	$x^2+y^2-8x-14y=-40$ $\Rightarrow(x-4)^2-16+(y-7)^2-49=-40$ $\Rightarrow(x-4)^2+(y-7)^2=25$ C_1: Radius = 5 and centre (4, 7) C_2: Radius = 7 and centre (16, 12) Distance between centres, d $d^2=(16-4)^2+(12-7)^2$ $=169$ $d=13$ Since the sum of the radii 5 + 7 = 12 is less than the distance between the centres, the circles do not intersect.
	Obtains correct equation	AO1.1b	A1	
	Obtains correct radius and correct coordinates of centre for C_1 Follow through 'their' equation	AO1.1b	A1F	
	States correct radius and correct coordinates of centre for C_2	AO1.1b	B1	
	Uses a method to find distance between the centres	AO3.1a	M1	
	Obtains correct distance	AO1.1b	A1	
	Demonstrates clearly that the sum of the radii of the two circles is less than the distance between 'their' centres and deduces that the circles do not overlap	AO2.2a	A1	
	Total		**7**	

4 (a) (Alt)	Attempts simultaneous equations and expands one bracket correctly	AO3.1a	M1	$x^2+y^2-8x-14y=-40$ $x^2+y^2-32x-24y=-351$ $24x+10y=311$ $y=\dfrac{311-24x}{10}$ $x^2+\left(\dfrac{311-24x}{10}\right)^2-8x-14\left(\dfrac{311-24x}{10}\right)=-40$ $x^2+\dfrac{96721-14928x+576x^2}{100}-8x-\dfrac{4354-336x}{10}+40=0$ $100x^2+96721-14928x+576x^2-800x-43540+3360x+4000=0$ $676x^2-12368x+57181=0$ $b^2-4ac=-1650000<0$ $\therefore$ no real roots so circles do not intersect
	Obtains correct equation	AO1.1b	A1	
	Eliminates squared terms	AO1.1b	A1f	
	Writes x in terms of y OE	AO1.1b	B1	
	Eliminates x or y to form quadratic	AO3.1a	M1	
	Obtains correct simplified quadratic	AO1.1b	A1	
	Demonstrates the quadratic has no real solutions and deduces that the circles do not intersect	AO2.2a	A1	
	Total		**7**	

(b)	Uses the fact that maximum distance is along the line of centres PI by a diagram	AO3.1a	M1	Max distance $= d+5+7$ Max distance $=25$
	Obtains maximum distance Follow through 'their' distance and radius for C_1	AO1.1b	A1	
	Total		**2**	

(a)	Uses a technique which could lead to showing two lines are perpendicular. Obtains at least one correct distance (or distance^2) or gradient.	AO3.1a	M1	$AB^2 = (8-15)^2 + (17-10)^2$ $= 98$ $AC^2 = (8--2)^2 + (17--7)^2$ $= 676$
	Obtains three correct distances (or distance^2) or two gradients. Lengths: $7\sqrt{2}, 17\sqrt{2}, 26$ Gradients: $AB = -\frac{7}{7}, BC = \frac{17}{17}$	AO1.1b	A1	$CB^2 = (15--2)^2 + (10--7)^2$ $= 578$ $AB^2 + BC^2 = 98 + 578$ $= 676$ $= AC^2$
	Completes correct rigorous argument to show required result Uses Pythagoras OR Multiplies gradients to show product is -1 AND Writes a concluding statement.	AO2.1	R1	Angle ABC is a right angle.
(b)(i)	Explains why AC is a diameter Must reference angle subtended by diameter (condone “angle in a semi-circle”) or give full explanation.	AO2.4	E1	The angle subtended by a diameter is 90° $\therefore AC$ must be a diameter of the circle
(b)(ii)	Deduces correct radius (or radius^2)	AO2.2a	B1	Radius $\frac{\sqrt{676}}{2} = 13$ Centre $\left(\frac{8-2}{2}, \frac{17-7}{2}\right) = (3, 5)$ Distance from centre to D $(3--8)^2 + (5--2)^2 = 11^2 + 7^2$ $= 170 > 169$ So D lies outside the circle.
	Obtains mid-point of diameter	AO1.1b	B1	
	Uses $D(-8, -2)$ to find the distance or (distance^2) from *their* centre OE	AO1.1a	M1	
	Completes rigorous argument by comparing $\sqrt{170} > 13$ (or $170 > 169$) to show that D lies outside the circle	AO2.1	R1	
	Total		**8**	